Analysis 2, Summer 2024

List 0

"Previous topics"

Basic algebra

1. Find the two values of r for which $r^2 - 2r - 15 = 0$. $\boxed{r_1 = -3,\ r_2 = 5}$

2. Find the two values of r for which $r^2 - 2r + 15 = 0$. $\boxed{r_1 = 1 + i\sqrt{14},\ r_2 = 1 - i\sqrt{14}}$

3. (a) Solve $\sqrt{y} = \sin(x)$ for y. $\boxed{y = (\sin(x))^2}$

   (b) Solve $\ln(y) = \sin(x)$ for y. $\boxed{y = e^{\sin(x)}}$

4. Solve $\ln(x) - 3 = 7t$ for x. $\boxed{x = e^{3+7t} \text{ or } x = (e^3)e^{7t}}$

5. Solve $\dfrac{-1}{2y^2} = C + \sqrt{x^2+1}$ for y. $\boxed{y = \dfrac{\pm 1}{\sqrt{C - 2\sqrt{x^2+1}}}}$

6. Solve $e^y = 9\sin(3t) - t^2 + C$ for y. $\boxed{y = \ln(9\sin(3t) - t^2 + C)}$

7. Find the value of C for which $\dfrac{1}{2} = \dfrac{-3}{1+C}$. $\boxed{C = -7}$

8. Find the real value of C for which $\sin(0) = 10Ce^0 - \dfrac{2}{25C^2}$. $\boxed{C = \dfrac{1}{5}}$

9. Find values of A and B such that

$$A \cdot (x+6) + B \cdot (x-2) = 2x - 6.$$

   $\boxed{A = \dfrac{-1}{4}, B = \dfrac{9}{4}}$

10. Find values of C_1 and C_2 such that both of these equations are true:

$$2C_1 + 2C_2e^0 - 3\sin(0) - 0\sin(0) + 0\cos(t) = 0,$$
$$2C_2e^0 - 3\cos(0) + -0\sin(0) - \sin(0) - 0\cos(0) + \cos(0) = 1.$$

    $\boxed{C_1 = -2, C_2 = 2}$

11. If $y(x) = \dfrac{-1}{\sqrt{C - 2\sqrt{x^2+1}}}$ and $y(0) = -1$, find the value of C. $\boxed{C = 3}$

Linear algebra

12. Calculate the length (also called magnitude or norm) of the vector $5\hat{\imath} + \hat{\jmath} + 5\hat{k}$.
    $\sqrt{5^2 + 1^2 + 5^2} = \boxed{\sqrt{51}}$

13. Calculate $\big|\,[2, -3]\,\big|$. $\sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \boxed{\sqrt{13}}$

14. Give a unit vector (that is, a vector of magnitude 1) that points in the same direction as the vector $\vec{v} = [15, 8] = \begin{bmatrix} 15 \\ 8 \end{bmatrix} = 15\hat{\imath} + 8\hat{\jmath}$. $\frac{1}{|\vec{v}|}\vec{v} = \boxed{\frac{15}{17}\hat{\imath} + \frac{8}{17}\hat{\jmath}}$

15. Give a unit vector that points in the same direction as $5\hat{\imath} - 2\hat{\jmath}$. $\boxed{\frac{5}{\sqrt{28}}\hat{\imath} - \frac{2}{\sqrt{29}}\hat{\jmath}}$

16. Calculate the dot product (also called scalar product) of the vectors $\vec{u} = [0, 1]$ and $\vec{v} = [-8, 5]$. $\boxed{5}$

17. If $|\vec{v}| = 8$ and $|\vec{w}| = 7$ and the angle between $\vec{v}$ and $\vec{w}$ is $120° = \frac{2}{3}\pi$, what is the value of $\vec{v} \cdot \vec{w}$? $(8)(7)\cos(\frac{2}{3}\pi) = (8)(7)(-\frac{1}{2}) = \boxed{-28}$

18. If $|\vec{v}| = 3$ and $|\vec{n}| = 16$ ...
    (a) ... and $\vec{v}$ points in the same direction as $\vec{n}$, what is the value of $\vec{v} \cdot \vec{n}$? $\boxed{48}$
    (b) ... and $\vec{v}$ is perpendicular to $\vec{n}$, what is the value of $\vec{v} \cdot \vec{n}$? $\boxed{0}$
    (c) ... and $\vec{v}$ points in the exact opposite direction as $\vec{n}$ (this is sometimes called "anti-parallel"), what is the value of $\vec{v} \cdot \vec{n}$? $\boxed{-48}$

19. If $|\vec{u}| = 1$ and $|\vec{v}| = 4$,
    (a) is it possible that $\vec{u} \cdot \vec{v} = 2\sqrt{3}$? $\boxed{\text{yes}}$
    (b) is it possible that $\vec{u} \cdot \vec{v} = 2$? $\boxed{\text{yes}}$
    (c) is it possible that $\vec{u} \cdot \vec{v} = -2$? $\boxed{\text{yes}}$
    (d) is it possible that $\vec{u} \cdot \vec{v} = 3.81$? $\boxed{\text{yes}}$
    (e) is it possible that $\vec{u} \cdot \vec{v} = 4.61$? $\boxed{\text{no}}$
    (f) is it possible that $\vec{u} \cdot \vec{v} = -\sqrt{17}$? $\boxed{\text{no}}$
    (g) is it possible that $\vec{u} \cdot \vec{v} = -\sqrt{7}$? $\boxed{\text{yes}}$

20. If $|\vec{u}| = 1$ and $|\vec{w}| = 7$, describe ALL possible values that $\vec{u} \cdot \vec{w}$ could have. $\boxed{\text{anything between } -7 \text{ and } 7}$

21. If $|\vec{u}| = 1$ and $\vec{n} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$,
    (a) what is the largest possible value that $\vec{u} \cdot \vec{n}$ could have? $\boxed{5}$
    (b) give an example of a vector $\vec{u}$ such that $\vec{u} \cdot \vec{n}$ has the value from part (a). $\boxed{\vec{u} = \left[\frac{-3}{5}, \frac{4}{5}\right]}$ is the only correct example.
    (c) give an example of a vector $\vec{u}$ such that $\vec{u} \cdot \vec{n} = 0$. There are two possibilities: $\boxed{\vec{u} = \left[\frac{4}{5}, \frac{3}{5}\right]}$ or $\boxed{\vec{u} = \left[\frac{-4}{5}, \frac{-3}{5}\right]}$

22. Write $\dfrac{5x+6}{x^2-6x+8} = \dfrac{5x+6}{(x-2)(x-4)}$ as a sum of partial fractions.
That is, find A and B such that

$$\frac{5x+6}{x^2-6x+8} = \frac{A}{x-2} + \frac{B}{x-4} = \boxed{\frac{-8}{x-2} + \frac{13}{x-4}}$$

23. Write $\dfrac{2x-6}{(x-2)(x+6)}$ as a sum of partial fractions. $\boxed{\dfrac{-1/4}{x-2}+\dfrac{9/4}{x+6}}$

24. Write $\dfrac{36}{x^3+9x^2+18x}$ as a sum of partial fractions. $\boxed{\dfrac{-4}{x+3}+\dfrac{2}{x+6}+\dfrac{2}{x}}$

Analysis 1

25. Give the derivative (with respect to t) of $y = 2e^{3t}+4\sin(5t)+6\cos(7t)+8t^9+10$.
$y' = \boxed{6e^{3t}+20\cos(5t)-42\sin(7t)+72t^8}$

26. If $y = x^9$, calculate $\dfrac{dy}{dx}+y'(x)+y'$. $9x^8+9x^8+9x^8 = \boxed{27x^8}$

27. If $y = 5\,e^x\sin(\sqrt{14}x)$, calculate $y''-2y'+15y$ and simplify your answer. $\boxed{0}$

28. Find all critical points of $f(x) = x^4-4x^3-8x^2+2$ and classify each one as a local minimum, local maximum, or neither.
$\boxed{x=-1 \text{ is a local min}, x=0 \text{ is a local max}, x=4 \text{ is a local min}}$

29. Find and classify the critical points of $f(x) = e^{x^2}(2x+3)$.
$\boxed{x=-1 \text{ is a local max}, x=-\frac{1}{2} \text{ is a local min}}$

30. If $f(3)=5$, $f'(3)=0$, and $f''(3)=2$, could $x=3$ be a local minimum of $f(x)$? $\boxed{\text{yes}}$ Could it be a local maximum? $\boxed{\text{no}}$

31. Find the following indefinite integrals.

(a) $\displaystyle\int 11\,x^4\,dx = \boxed{\frac{11}{5}x^5+C}$

(b) $\displaystyle\int x^{-1/2}\,dx = \boxed{2x^{1/2}+C}$ or $\boxed{2\sqrt{x}+C}$

(c) $\displaystyle\int (\sin(2x))^2\cos(2x)\,dx = \boxed{\tfrac{1}{6}(\sin(2x))^3+C}$ using substitution $u=\sin(2x)$

(d) $\displaystyle\int \frac{x^4}{\sqrt{x^5+1}}\,dx = \boxed{\tfrac{2}{5}\cdot\sqrt{x^5+1}+C}$ using substitution $u=x^5+1$

(e) $\displaystyle\int 11\,y^4\,dy = \boxed{\frac{11}{5}y^5+C}$

(f) $\displaystyle\int \frac{1}{y^3}\,dy = \boxed{\frac{-1}{2y^2}+C}$

(g) $\displaystyle\int \frac{1}{y^2}\,dy = \boxed{\frac{-1}{y}+C}$

(h) $\displaystyle\int \frac{1}{y}\,dy = \boxed{\ln(y)+C}$

(i) $\int e^{6t}\,\mathrm{d}t = \boxed{\frac{1}{6}e^{6t} + C}$

(j) $\int te^{t}\,\mathrm{d}t = \boxed{t\,e^{t} - e^{t} + C}$ using "parts"

(k) $\int e^{-x}(2x-3)\,\mathrm{d}x = \boxed{(1-2x)e^{-x} + C}$ using "parts"

32. Find the definite integral $\int_0^1 (4x^3 - 9x^2)\,\mathrm{d}x$. (Your answer should be a number.) $\boxed{-2}$

33. Give the definite integral $\int_0^1 (4x^3 - 9x^2k^2)\,\mathrm{d}x$. (Your answer should be a formula with k.) $\boxed{1 - 3k^2}$

34. Calculate $\int_a^b x\,\mathrm{d}x$. (Your answer should be a formula with a and b.) $\boxed{\frac{1}{2}b^2 - \frac{1}{2}a^2}$

35. Calculate $\int_{q^2}^{\sin q} x\,\mathrm{d}x$. (Your answer should be a formula with q.) $\boxed{\frac{1}{2}(\sin q)^2 - \frac{1}{2}q^4}$

36. Calculate (a) $\int_0^3 xe^{2x}\,\mathrm{d}x$, (b) $\int_0^3 te^{2t}\,\mathrm{d}t$, (c) $\int_0^3 ye^{2y}\,\mathrm{d}y$. All are $\boxed{\frac{1+5e^{6t}}{4}}$.